الامتحان الوطني الموحد للبكالوريا

المسالك الدولية

الدورة العادية 2021

- الموضوع –

SSSSSSSSSSSSSSSSSSS | NS 24E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

المادة	الرياضيات	مدة الإنجاز	4h
الشعبة أو المسلك	شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية)	المعامل	9

- The duration of the exam is four hours

-The exam consists of 3 independent exercises

- The exercises can be treated in the order chosen by the candidate

- Exercise1 concerns analysis ……..……(12 pts)

- Exercise2 concerns complex numbers.....…..(4 pts)

- Exercise3 concerns arithmetic ……..(4 pts)

The use of the calculator is not allowed

The use of red color is not permitted

EXERCISE1 : (12 points)

For every natural number n we consider the function f_n defined on $\mathbb{R}$ by :

$$f_n(x)=\frac{-2e^x}{1+e^x}+nx$$

And let (C_n) be its representetive curve in an orthonormal coordinate system $(O,\vec{i},\vec{j})$.

(We will take $\|\vec{i}\|=\|\vec{j}\|=1cm$)

Part I :

0.5 1-a) Calculate $\lim\limits_{x\to+\infty}\left(f_n(x)-nx+2\right)$, then interpret graphically the obtained result.

0.5 b) Show that the curve (C_n) admits, at $-\infty$, an asymptote (Δ_n) for which you will determine a cartesian equation .

0.5 2-a) Show that the function f_n is differentiable on $\mathbb{R}$ and that :

$$(\forall x\in\mathbb{R})\ ;\ f_n'(x)=\frac{-2e^x}{\left(1+e^x\right)^2}+n$$

0.5 b) Show that : $(\forall x\in\mathbb{R})\ ;\ \frac{4e^x}{\left(1+e^x\right)^2}\le 1$

0.5 c) Deduce the sense of variation of the function f_n on $\mathbb{R}$

(We distinguish the two cases : $n=0$ and $n\ge 1$)

0.5 3-a) Determine the equation of the tangent to (C_n) at the point I with abscissa 0

0.5 b) Show that the point I is the unique inflection point of the curve (C_n)

0.5 4- Plot in the same coordinate system, the curves (C_0) and (C_2).

5- For all real number $t>0$, $A(t)$ denotes the area of the region delimited by the curve (C_n) and the lines with equations : $y=nx-2$, $x=0$ and $x=t$

0.5 a) Calculate $A(t)$ for every $t>0$

0.5 b) Calculate $\lim\limits_{t\to+\infty}A(t)$

Part II :

Consider the numerical sequence $(u_n)_{n\ge 0}$ defined by :

$u_0=0$ and $(\forall n\in\mathbb{N})\ ;\ u_{n+1}=f_0(u_n)$

0.5 1-a) Show that the equation $f_0(x)=x$ admits a unique solution α in $\mathbb{R}$

0.5 b) Show that : $(\forall x\in\mathbb{R})\ ;\ |f_0'(x)|\le\frac{1}{2}$

0.5 2-a) Show that : $(\forall n\in\mathbb{N})\ ;\ |u_{n+1}-\alpha|\le\frac{1}{2}|u_n-\alpha|$

0.5 b) Deduce that : $(\forall n \in \mathbb{N})$; $|u_n - \alpha| \leq \left(\frac{1}{2}\right)^n |\alpha|$

0.5 c) Show that the sequence $(u_n)_{n\geq 0}$ converges to α .

Part III :

Suppose in this part that $n \geq 2$

0.5 1- a) Show that for all natural number $n \geq 2$, there exists a unique real number x_n which is a solution of the equation $f_n(x) = 0$

0.5 b) Show that for all natural number $n \geq 2$, $0 < x_n < 1$

(we will take $\frac{2e}{1+e} < 1.47$)

0.5 2-a) Show that for all natural number $n \geq 2$, $f_{n+1}(x_n) > 0$

0.5 b) Deduce that the sequence $(x_n)_{n\geq 2}$ is strictly decreasing.

0.5 c) Show that the sequence $(x_n)_{n\geq 2}$ is convergent.

0.5 3-a) Sow that for all natural number $n \geq 2$, $\frac{1}{n} < x_n < \frac{1}{n}\left(\frac{2e}{1+e}\right)$

0.5 b) Deduce $\lim\limits_{n\to+\infty} x_n$ then show that : $\lim\limits_{n\to+\infty} nx_n = 1$

0.5 4-a) Show that for all natural number $n \geq 2$, we have $x_n \leq x_2$

0.5 b) Deduce $\lim\limits_{n\to+\infty} (x_n)^n$

EXERCISE2 : (4 points)

Let a, b and c be three complex numbers differing zero such that : $a+b \neq c$

0.5 1-a) Solve in $\mathbb{C}$ the equation with variable z (E) : $z^2 - (a+b+c)z + c(a+b) = 0$

0.5 b) We suppose in this question that : $a = i$, $b = e^{i\frac{\pi}{3}}$ and $c = a - b$

Write the solutions of the equation (E) in exponential form .

2- The complex plane is attached to a direct orthonormal coordinate system $(O, \vec{u}, \vec{v})$.

Consider the three points $A(a)$, $B(b)$ and $C(c)$ assumed non-collinear.

Let $P(p)$ be the center of the rotation with angle $\frac{\pi}{2}$ which sends B to A

And $Q(q)$ be the center of the rotation with angle $\left(-\frac{\pi}{2}\right)$ which sends C to A

And $D(d)$ be the midpoint of the segment $[BC]$

1 a) Show that : $2p = b + a + (a-b)i$ and $2q = c + a + (c-a)i$

0.5 b) Calculate : $\dfrac{p-d}{q-d}$

0.5 c) Deduce the nature of the triangle PDQ

3- Let E be the symmetric point of B with respect to P, and F be the symmetric point of C with respect to Q and K be the midpoint of the segment $[EF]$.

0.5 a) Show that the affix of K is $k = a + \dfrac{i}{2}(c-b)$

0.5 b) Show that the points K , P , Q and D are cocyclic.

EXERCISE3 : (4 points)

Part I : Consider in $\mathbb{Z}\times\mathbb{Z}$ the equation (E) : $47x - 43y = 1$

0.25 1- Verify that the couple $(11,12)$ is a particular solution of the equation (E)

0.75 2- Solve in $\mathbb{Z}\times\mathbb{Z}$ the equation (E)

Part II : Consider in $\mathbb{Z}$ the equation (F) : $x^{41} \equiv 4 \ [43]$

1- Let x be a solution of the equation (F)

0.5 a) Show that x and 43 are coprime, then deduce that : $x^{42} \equiv 1 \ [43]$

0.5 b) Show that : $4x \equiv 1 \ [43]$, then deduce that : $x \equiv 11 \ [43]$

0.5 2- Give the set of the solutions in $\mathbb{Z}$ of the equation (F)

Part III : Consider in $\mathbb{Z}$ the system with two equations (S) : $\begin{cases} x^{41} \equiv 4 \ [43] \\ x^{47} \equiv 10 \ [47] \end{cases}$

1- Let x be a solution of the system (S)

0.5 a) Show that x is a solution of the system (S') : $\begin{cases} x \equiv 11 \ [43] \\ x \equiv 10 \ [47] \end{cases}$

0.5 b) Deduce that : $x \equiv 527 \ [2021]$ (One can use Part I)

0.5 2- Give the set of the solutions in $\mathbb{Z}$ of the system (S)

END