الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة الاستدراكية 2020
- الموضوع –

SSSSSSSSSSSSSSSSSS | RS 24E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية)	المعامل	9

- The duration of the exam is 4 hours.
- The exam is made up of (5) pages numbered from 1/5 to 5/5
- The exam includes four independent exercises.
- **The candidate must treat EXERCISE3 and EXERCISE4, and choose to treat EXERCISE1 or EXERCISE2**
- **The candidate must perform totally three (3) exercises:**

- **EXERCISE1** which concerns arithmetic **(optional)**..... **3.5 points**
  or
  EXERCISE2 which concerns algebraic structures **(optional)**...**3.5 points**

- **EXERCISE3** which concerns complex numbers **(obligatory)** .....**3.5 points**

- **EXERCISE4** which concerns analysis **(obligatory)** .....**13 points**

The use of the calculator is not allowed

choose to treat EXERCISE1 or EXERCISE2
and treat obligatory EXERCISE3 and EXERCISE4

EXERCISE1 :(3.5 points/optional)

If you choose to treat exercise1 you do not treat exercise2

Let p and q be two prime numbers satisfying: $p < q$ and $9^{p+q-1} \equiv 1 \ [pq]$

0.5 1-a) Show that p and 9 are coprime.

1 b) Deduce that: $9^{p-1} \equiv 1 \ [p]$ and that $9^{q} \equiv 1 \ [p]$

0.5 2-a) Show that $p-1$ and q are coprime.

0.5 b) Using the BEZOUT's theorem, show that: $p = 2$

0.5 3-a) Using the FERMAT's theorem, show that: $9^{q-1} \equiv 1 \quad [q]$

0.5 b) Deduce that: $q = 5$

EXERCISE2 : (3.5 points/optional)

If you choose to treat exercise2 you do not treat exercise1

$M_3(\mathbb{R})$ denotes the set of the 3×3 matrices with real entries.

Recall that $(M_3(\mathbb{R}),+,\times)$ is a real vector space with dimension 9 and that $(M_3(\mathbb{R}),+,\cdot)$ is a unitary non-commutative ring with zero $O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and unit $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Consider the sub-set: $E = \left\{ M(x,y,z) = \begin{pmatrix} x & -y & -y \\ 0 & z & 0 \\ y & x-z & x \end{pmatrix} / \ (x,y,z) \in \mathbb{R}^3 \right\}$

First part :

0.25 1- a) Show that E is sub-space of the real space $(M_3(\mathbb{R}),+,\times)$

0.5 b) Determine a basis of $(E,+,\times)$

0.25 2- a) Verify that:

$$\forall (x,y,z) \in \mathbb{R}^3, \forall (x',y',z') \in \mathbb{R}^3 ; M(x,y,z) \cdot M(x',y',z') = M(xx'-yy', xy'+yx', zz')$$

0.5 b) Show that $(E,+,\cdot)$ is a commutative ring

Second part :

Consider the sub-set F of E containing the matrices of the form $M(x,y,0)$ where $(x,y) \in \mathbb{R}^2$

0.25 1- Show that F is a sub-group of the group $(E,+)$

2- φ denotes the map from $\mathbb{C}^*$ to E defined by :

$$\forall (x,y) \in \mathbb{R}^2 \ ; \quad \varphi(x+iy) = M(x,y,0)$$

0.25 a) Show that φ is an homomorphism from $(\mathbb{C}^*,\cdot)$ to $(E,\cdot)$

0.5 b) Deduce that $(F^*,\cdot)$ is a commutative group (F^* denotes $F - \{O\}$)

0.5 c) Show that $(F,+,\cdot)$ is a commutative field whose unit must be specified.

0.25 3- a) Verify that: $(\forall M(x,y,0) \in F) \quad ; \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} M(x,y,0) = O$

0.25 b) Deduce that no element of the sub-set F admits an inverse for the multiplication in $M_3(\mathbb{C})$

EXERCISE3 :(3.5 points/obligatory)

I- Let m be a non-zero real number.

Consider in the set of complex numbers $\mathbb{C}$, the two equations with variable z:

$(E): z^2 + 2z + 1 + m^2 = 0$ and $(F): z^3 + 2(1-i)z^2 + (1+m^2-4i)z - 2i(1+m^2) = 0$

0.5 1- Solve in $\mathbb{C}$ the equation (E)

0.25 2- a) Show that the equation (F) admits a pure imaginary solution which must be determined.

0.5 b) Solve in $\mathbb{C}$ the equation (F)

II- The complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$

Consider the two points: $A(-1+im)$ and $B(-1-im)$

Let W be the midpoint of the segment $[AB]$, and A' be the midpoint of the segment $[OB]$ and B' the midpoint of the segment $[OA]$

The rotation with center W and angle $\frac{\pi}{2}$ sends A to $P(p)$, and the rotation with center A' and angle $\frac{\pi}{2}$ sends B to $Q(q)$ and the rotation with center B' and angle $\frac{\pi}{2}$ send O to $R(r)$

1.5 1- Show that $p = -1+m$, $q = \frac{1-i}{2}(-1-im)$ and $r = \bar{q}$

0.25 2- a) Verify that $q - r = -ip$

0.5 b) Deduce that $OP = QR$ and that the lines (OP) and (QR) are perpendicular.

EXERCISE4 : (13 points/obligatory)

First part :

Consider the numerical function f defined on $I = [0,1]$ by $f(x) = x\ln(2-x)$

and let (C) be its curve in an orthonormal coordinate system $(O;\vec{i},\vec{j})$.

0.75 1-a) Show that f is differentiable on I and that : $\forall x \in I \; ; \quad f'(x) = \ln(2-x) - \frac{x}{2-x}$

0.5 b) Show that the differentiable function f' is strictly decreasing on I

c) Show that there exists a unique real number $a \in]0;1[$ such that : $f'(a) = 0$

0.75 and that $f(a) = \frac{a^2}{2-a}$

0.75 2-a) Study the variations of f, and then set up its table of variations.

0.5 b) Show that the curve (C) is concave.

0.5 c) Show that $(\forall t \in I)$, $(\forall x \in I)$; $f(x) \geq f'(t)(x-t) + f(t)$

0.5 d) Deduce that $(\forall x \in I)$; $f(x) \geq x \ln 2$ and $f(x) \geq -x + 1$

0.5 3- Plot the curve (C) (we take: $\|\vec{i}\| = 2cm$)

0.75 4- Calculate, in cm^2, the area of the region bounded by the curve (C) and the lines with the following equations : $x = 0$, $x = 1$ and $y = 0$

Second part :

Let n be a natural number greater or equal to 2.

Consider the numerical function f_n defined on $I = [0,1]$ by : $f_n(x) = x^n \ln(2-x)$

0.5 1-a) Verify that f_n is positive on I and that $f_n(0) = f_n(1)$

0.5 b) Show that there exists at least an $a_n \in]0,1[$ such that : $f_n'(a_n) = 0$

0.75 2- a) Show that f_n is differentiable on I and that : $\forall x \in I; \quad f_n'(x) = x^{n-1} g_n(x)$ where :

$g_n(x) = n \ln(2-x) - \frac{x}{2-x}$

0.5 b) Show that the function g_n is strictly decreasing on I

0.5 c) Deduce that a_n is unique.

3- Consider the sequence $(a_n)_{n \geq 2}$ as defined above.

1 a) Show that : $\forall n \geq 2; \quad f_n(a_n) = \frac{1}{n} \cdot \frac{a_n^{n+1}}{2-a_n}$, and then deduce that $\lim\limits_{n \to +\infty} f_n(a_n) = 0$

1 b) Show that : $\forall n \geq 2 \quad g_n(a_{n+1}) = -\ln(2-a_{n+1})$,and then deduce that the sequence $(a_n)_{n \geq 2}$ is strictly increasing.

0.25 c) Show that the sequence $(a_n)_{n \geq 2}$ is convergent.

0.5 d) Show that $\lim\limits_{n\to+\infty} a_n = 1$

Third part :

For each natural number $n \geq 2$, we put: $I_n = \int_0^1 f_n(x)dx$

0.75 1- Show that the sequence $(I_n)_{n\geq 2}$ is decreasing, and then deduce that it converges.

0.5 2- Using an integration by parts show that: $I_n = \frac{1}{n+1}\int_0^1 \frac{x^{n+1}}{2-x}dx$

0.75 3- Show that: $(\forall n \geq 2)$; $0 \leq I_n \leq \frac{1}{n+1}$, and then deduce that $\lim\limits_{n\to+\infty} I_n = 0$

END

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