الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة العادية 2020
- الموضوع –

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

SSSSSSSSSSSSSSSSSS | NS 24E

المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية)	المعامل	9

- The duration of the exam is 4 hours.
- The exam is made up of (5) pages numbered from 1/5 to 5/5
- The exam includes four independent exercises.
- **The candidate must treat the EXERCISE3 and EXERCISE4, and choose to treat**

  EXERCISE1 or EXERCISE2
- **The candidate must perform totaly three (3) exercises:**

- **EXERCISE1** which concerns arithmetic **(optional)**.....**3.5 points**
- **or**

EXERCISE2 which concerns algebraic structures **(optional)**.....**3.5 points**

- EXERCISE3 which concerns complex numbers **(obligatory)** .....**3.5 points**

- EXERCISE4 which concerns analysis **(obligatory)** .....**13 points**

The use of the calculator is not allowed

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choose to treat EXERCISE1 or EXERCISE2
and treat obligatory EXERCISE3 and EXERCISE4

EXERCISE1 :(3.5 points/optional)

(If you choose to treat EXERCISE1, you do not treat EXERCISE2)

Consider in $\mathbb{Z} \times \mathbb{Z}$ the equation (D) : $7x^3 - 13y = 5$

1- Let $(x,y)\in \mathbb{Z}\times\mathbb{Z}$ be a solution of the equation (D)

0.5 a) Show that x and 13 are coprime.

0.5 b) Deduce that : $x^{12}\equiv 1\ [13]$

1 c) Show that : $x^3\equiv 10\ [13]$

0.5 d) Deduce that : $x^{12}\equiv 3\ [13]$

1 2- Deduce from the previous questions, that the equation (D) does not admit a solution in $\mathbb{Z}\times\mathbb{Z}$

EXERCISE2 : (3.5 points/optional)

(If you choose to treat EXERCISE2, you do not treat EXERCISE1)

$M_2(\mathbb{R})$ denotes the set of square matrices of order two.

We recall that $(M_2(\mathbb{R}),+,\times)$ is a non commutative ring with unit $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ and that $(\mathbb{R}^*,\times)$ is a commutative group.

Consider the sub-set E of $M_2(\mathbb{R})$ defined by : $E=\left\{\begin{pmatrix}1&x\\0&y\end{pmatrix}/\ x\in\mathbb{R}\ \text{and}\ y\in\mathbb{R}^*\right\}$.

0.5 1- a) Show that E is a stable part of $(M_2(\mathbb{R}),\times)$

0.5 b) Show that the multiplication is not commutative in E

0.5 c) Verify that : $(\forall x\in\mathbb{R})(\forall y\in\mathbb{R}^*)\ ;\ \begin{pmatrix}1&x\\0&y\end{pmatrix}\times\begin{pmatrix}1&-\frac{x}{y}\\0&\frac{1}{y}\end{pmatrix}=\begin{pmatrix}1&-\frac{x}{y}\\0&\frac{1}{y}\end{pmatrix}\times\begin{pmatrix}1&x\\0&y\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

0.5 2-Show that $(E,\times)$ is a non commutative group.

3- Consider the sub-set F of E defined by : $F=\left\{M(x)=\begin{pmatrix}1&x-1\\0&x\end{pmatrix}/\ x\in\mathbb{R}^*\right\}$

0.5 a) Show that the map φ defined by $(\forall x\in\mathbb{R}^*)$; $\varphi(x)=M(x)$ is an homomorphism from $(\mathbb{R}^*,\times)$ to $(E,\times)$.

1 b) Deduce that $(F,\times)$ is a commutative group whose neutral element will be specified.

EXERCISE3 :(3.5 points/obligatory)

Let m be a non-zero complex number.

First part :

Consider in $\mathbb{C}$ the equation with variable z, (E) : $z^3-2mz^2+2m^2z-m^3=0$

0.5 1- Solve in $\mathbb{C}$ the equation (E) (notice that m is a solution of the equation (E))

2- z_1 and z_2 denote the other two solutions of the equation (E), which are different from m

0.25 a) Verify that: $\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{m}$

0.5 b) In case where $m=1+e^{i\frac{p}{3}}$, write in algebrac form z_1 and z_2

Second part :

The complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$

Consider the points A and B with respective affixes $a=me^{i\frac{p}{3}}$ and $b=me^{-i\frac{p}{3}}$

We denote P the center of the rotation with angle $\frac{p}{2}$ which sends O to A

and Q the center of the rotation with angle $\frac{p}{2}$ which sends A to B

and R the center of the rotation with angle $\frac{p}{2}$ which sends B to O

0.25 1- Show that the points O, A and B are non collinear.

1 2-a) Show that the affix of P is $p=m\frac{\sqrt{2}}{2}e^{i\frac{7p}{12}}$ and that the affix of R is $r=m\frac{\sqrt{2}}{2}e^{-i\frac{7p}{12}}$

0.5 b) Show that the affix of Q is $q=m\sqrt{2}\sin\frac{7p}{12}$

0.5 3-Show that $OQ=PR$ and that the lines (OQ) and (PR) are perpendicular.

EXERCISE4 :(13 points/obligatory)

First part :

Consider the numerical function f defined on the interval $I=[0;+\infty[$ by :

$$f(0)=0 \quad \text{and} \quad \forall x\in]0;+\infty[, \quad f(x)=x^3\ln\left(1+\frac{1}{x}\right)$$

Let (C) be its curve in an orthonormal coordinate system $(O;\vec{i},\vec{j})$

(We will take $\|\vec{i}\|=\|\vec{j}\|=1cm$)

0.5 1- By using the mean values theorem to the function t a $\ln(t)$ on the interval $[x;x+1]$,

show that : (P) $(\forall x\in]0;+\infty[)$; $\frac{1}{x+1}<\ln\left(1+\frac{1}{x}\right)<\frac{1}{x}$

0.5 2- a) By using the proposition (P) show that the function is differentiable on the right at 0

0.5 b) By using the proposition (P) show that the curve (C) admits a parabolic branch whose direction must be specified.

0.75 3-a) Show that the fonction f is differentiable on $]0;+\infty[$ and :

$$(\forall x \in]0;+\infty[) \quad ; \quad f'(x)= 3x^2\left(\ln\left(1+\frac{1}{x}\right)-\frac{1}{3(1+x)}\right)$$

0.5 b) Deduce that the function f is strictly increasing on I (one can use the propostion (P))

0.25 c) Set the table of variations of f

4- For $x \in]0;+\infty[$,we put : $g(x)= \dfrac{f(x)}{x}$

0.75 a) Verify that : $(\forall x \in]0;+\infty[) \quad ; \quad g'(x)= 2x\left(\ln\left(1+\frac{1}{x}\right)-\frac{1}{2(1+x)}\right)$, and then deduce that the function g is strictly increasing on $]0;+\infty[$

0.5 b) Show that the equation $g(x)= 1$ admits in $]0;+\infty[$, a unique solution denoted a and verify that $a \in]1;2[$ (We will take $\ln 2 = 0.7$ and $\ln\dfrac{3}{2} = 1.5$)

0.5 c) Deduce that the only solutions of the equation $f(x)= x$ are 0 and a

0.5 5-a) Plot the curve (C).

(We will specify the right half-tangent at O and the parabolic branch of (C))

0.25 b) Show that f is a bijection from I to I (f^{-1} denotes its inverse bijection)

Seconde part :

We consider the numerical sequence $(u_n)_{n\geq 0}$ defined by :

$$0 < u_0 < a \quad \text{and} \quad (\forall n \in \mathbb{N}) \; ; \; u_{n+1} = f^{-1}(u_n)$$

0.5 1-Show by induction that : $(\forall n \in \mathbb{N}) \quad ; \quad 0 < u_n < a$

0.5 2-a) Show that $g([0;a]) = [0;1[$

0.5 b) Deduce that the sequence $(u_n)_{n\geq 0}$ is strictly increasing.

0.25 c) Show that the sequence $(u_n)_{n\geq 0}$ is convergent.

0.5 3- Determine $\lim\limits_{n\to+\infty} u_n$

Third part :

We consider the function F defined on the interval I by : $(\forall x \in I)\ ;\ F(x)=\int_x^1 f(t)dt$

0.5 1-a) Study according to the values of x, the sign of $f(x)$

0.5 b) Show that the function F is differentiable on I and determine its differentiable Function F'

0.25 c) Deduce that F is strictly decreasing on I

0.5 2-a) Show that : $(\forall x \in [1;+\infty[)\ ;\ F(x) \leq (1-x)\ln 2$

0.25 b) Deduce $\lim\limits_{x \to +\infty} F(x)$

0.5 3-a) By using an integration by parts show that :

$$\forall x \in]0;+\infty[\ ;\ F(x)=\frac{\ln 2}{4}-\frac{x^4}{4}\ln\left(1+\frac{1}{x}\right)+\frac{1}{4}\int_x^1 \frac{t^3}{t+1}dt$$

0.5 b) Calculate $\int_x^1 \frac{t^3}{t+1}dt$ for all $x \in]0;+\infty[$ (notice that: $\frac{t^3}{1+t}=t^2-t+1-\frac{1}{1+t}$)

0.5 c) Deduce that : $\forall x \in]0;+\infty[\ ;\ F(x)=\frac{5}{24}-\frac{x^3}{12}+\frac{x^2}{8}-\frac{x}{4}+\frac{1}{4}\ln(1+x)-\frac{x^4}{4}\ln\left(1+\frac{1}{x}\right)$

0.5 d) Calculate $\lim\limits_{x \to 0^+} F(x)$, and then deduce the value of $\int_0^1 f(t)dt$

4-For any non-zero natural number n , we put: $v_n=\sum_{k=0}^{k=n-1}\left(F\left(\frac{2k+1}{2n}\right)-F\left(\frac{k}{n}\right)\right)$

0.5 a) Show that for all $n \in \mathbb{N}^*$ and for all $k \in \{0,1,.....,n-1\}$:

$$-\frac{1}{2n}f\left(\frac{2k+1}{2n}\right) \leq F\left(\frac{2k+1}{2n}\right)-F\left(\frac{k}{n}\right) \leq -\frac{1}{2n}f\left(\frac{k}{n}\right)$$

0.5 b) Deduce that: $(\forall n \in \mathbb{N}^*)\ ;\ -\frac{1}{2n}\sum_{k=1}^{k=n} f\left(\frac{k}{n}\right) \leq v_n \leq -\frac{1}{2n}\sum_{k=0}^{k=n-1} f\left(\frac{k}{n}\right)$

(notice that : $\frac{2k+1}{2n} < \frac{k+1}{n}$)

0.25 c) Show that the numerical sequence $(v_n)_{n \in \mathbb{N}^*}$ is convergent and determine its limit.

END