الامتحان الوطني الموحد للبكالوريا
المسالك الدولية – خيار أنجليزية
الدورة الاستدراكية 2019
- الموضوع -

RS24E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي

المركز الوطني للتقويم والامتحانات والتوجيه

المادة	الرياضيات	مدة الانجاز	4
الشعبة أو المسلك	شعبة العلوم الرياضية : (أ) و (ب) – خيار أنجليزية	المعامل	9

- The duration of the exam is 4 hours.
- The exam contains 4 independent exercises.
- The exercises can be treated in the order chosen by the candidate.

- Exercise 1 concerns the complex numbers .....(3.5 pts)
- Exercise 2 concerns the caculus of probabilities.....(3 pts)
- Exercise 3 concerns the analysis .....(3.5 pts)
- Exercise 4 concerns the analysis.....(10 pts)

The use of calculators is not allowed
The use of red pens on answers sheets is not permitted

EXERCISE1 :(3.5 points)

Let α be a complex number different from zero.

I- Consider in the set of complex numbers $\mathbb{C}$ the equation of variable z :

(E_α) : $z^2 - i\alpha\sqrt{3}z - \alpha^2 = 0$

0.25 1-a- Verify that the discriminant of (E_α) is $\Delta = \alpha^2$

0.5 b- Solve in $\mathbb{C}$ the equation (E_α)

0.5 2- Knowing that $\alpha = |\alpha| e^{i\lambda}$ ($\lambda \in \mathbb{R}$), rewrite the two solutions of the equation (E_α) in exponential polar form .

II- Suppose that the complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$.

Consider the points Ω , M_1 and M_2 with respective affixes α , $z_1 = \frac{1+i\sqrt{3}}{2}\alpha$ and $z_2 = \frac{-1+i\sqrt{3}}{2}\alpha$ and let R be the rotation with center O and angle $\frac{\pi}{3}$

0.5 1-a- Show that $R(\Omega) = M_1$ and $R(M_1) = M_2$

0.25 b- Deduce that the triangles $O\Omega M_1$ and OM_1M_2 are equilateral.

0.25 2-a- Verify that : $z_1 - z_2 = \alpha$

0.5 b- Show that the lines (ΩM_2) and (OM_1) are perpendicular.

0.25 c- Deduce that $O\Omega M_1M_2$ is a rhombus.

0.5 3- Show that for each real number θ , the number : $Z = \frac{z_2 - \alpha}{z_1 - \alpha} \div \frac{z_2 - |\alpha| e^{i\theta}}{z_1 - |\alpha| e^{i\theta}}$ is a real number.

EXERCISE2:(3 points)

An urn contains n ($n \in \mathbb{N}^*$, $n \geq 3$) balls indistinguishable by touch and numbered from 1 to n. We draw at random all the balls successively one by one without replacing.

1 1- What is the probability that the balls numbered 1, 2 and 3 are drawn successively in this order ?

1 2- What is the probability that the balls 1, 2 and 3 are drawn in this order but successively or not ?

1 3- Consider the random variable X_n equal to the number of necessary draws for selecting the balls 1 ,2 and 3.

Give the probability law (distribution) for the random variable X_n

EXERCISE3 : (3.5 points)

Consider the real vector space $(V_2,+,.)$ with dimension 2 .

Let $(\vec{i},\vec{j})$ be a basis of V_2. Put : $\vec{e_1} = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j}$ and $\vec{e_2} = \frac{1}{2}\vec{i} - \frac{1}{2}\vec{j}$

Let $*$ be the law internal composition defined in V_2 by :

$\forall (x,y,x',y') \in \mathbb{R}^4 \quad (x\vec{i} + y\vec{j}) * (x'\vec{i} + y'\vec{j}) = (xx' + yy')\vec{i} + (xy' + yx')\vec{j}$

0.25 1-a- Show that $(\vec{e_1},\vec{e_2})$ is a basis of V_2

0.25 b-Verify that : $\vec{e_1}*\vec{e_1}=\vec{e_1}$; $\vec{e_2}*\vec{e_2}=\vec{e_2}$ and $\vec{e_1}*\vec{e_2}=\vec{e_2}*\vec{e_1}=\vec{0}$

0.25 c- Show that : $\forall(X,X',Y,Y')\in\mathbb{R}^4\ \ (X\vec{e_1}+Y\vec{e_2})*(X'\vec{e_1}+Y'\vec{e_2})=XX'\vec{e_1}+YY'\vec{e_2}$

0.25 2- a- Show that $*$ is commutative.

0.25 b- Show that $*$ is associative.

0.25 c- Show that $*$ posses an identity element.

0.25 d- Show that $(V_2,+,*)$ is an unitary commutative ring.

3- Let $\vec{u}\in V_2-\{\vec{0}\}$. Put : $E_{\vec{u}}=\{\lambda\vec{u}/\lambda\in\mathbb{R}\}$

0.25 a- Show that $(E_{\vec{u}},+)$ is a sub-group of the group $(V_2,+)$

0.25 b- Show that $(E_{\vec{u}},+,.)$ is a sub-space of the real vector space $(V_2,+,.)$

0.5 c- Show that : $E_{\vec{u}}$ is stable for $*\Leftrightarrow$ $\vec{u}*\vec{u}$ *and* $\vec{u}$ *are collinear*

4- Suppose : $(\exists\alpha\in\mathbb{R})\ ;\vec{u}*\vec{u}=\alpha\vec{u}$

Consider the mapping $\varphi:\ \mathbb{R}^*\longrightarrow E_{\vec{u}}$

$$x\ \mapsto\ \frac{x}{\alpha}\vec{u}$$

0.5 a- Show that φ is an isomorphism from $(\mathbb{R}^*,\times)$ to $(E_{\vec{u}},*)$

0.25 b- Deduce that $(E_{\vec{u}},+,*)$ is a commutative field.

EXERCISE4 : (10 points)

PART I

Consider the function g defined on $I=]-1,+\infty[$ by : $g(x)=1+x^2-2x(1+x)\ln(1+x)$

0.25 1- a- Show that : $\lim\limits_{x\to-1^+}g(x)=2$

0.5 b- Show that : $\lim\limits_{x\to+\infty}g(x)=-\infty$

0.5 2- Show that g is differentiable on I and that $(\forall x\in I)\quad g'(x)=-2(1+2x)\ln(1+x)$

3- The table of variations of the function g is given bellow:

x	-1		$-\frac{1}{2}$		0		$+\infty$
$g'(x)$	‖	-	0	+	0	-	
$g(x)$	2	↘	$\frac{5}{4}-\frac{\ln 2}{2}$	↗	1	↘	$-\infty$

0.5 a- Show that there exists a unique strictly positive real number α such that : $g(\alpha)=0$

0.25 b- Verify that : $\alpha < 1$ (We take : $\ln 2 = 0.7$)

0.5 c- Deduce that : $(\forall x \in]-1,\alpha[) \quad 0 < g(x)$ and that : $(\forall x \in]\alpha,+\infty[) \quad g(x) < 0$

Part II : Consider the function f defined on $I =]-1,+\infty[$ by : $f(x) = \dfrac{\ln(1+x)}{1+x^2}$

and let (C) be its representative curve in an orthonormal coordinate system $(O,\vec{i},\vec{j})$.

0.5 1-a- Calculate $\lim\limits_{x \to -1^+} f(x)$ and then interpret graphically the obtained result.

0.5 b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and then interpret graphically the obtained result.

0.75 2- a- Show that f is differentiable on I and that $(\forall x \in I) \quad f'(x) = \dfrac{g(x)}{(1+x)(1+x^2)^2}$

0.5 b- Give the variations of f on I

0.75 c- Verify that : $f(\alpha) = \dfrac{1}{2\alpha(1+\alpha)}$ and that : $(\forall x \in I) \quad f(x) \le \dfrac{1}{2\alpha(1+\alpha)}$

0.25 3-a- Give the equation of the tangente (T) to (C) at the point with abscissa 0

0.5 b- Show that : $(\forall x > 0) \quad \ln(1+x) < x$

0.25 c- Deduce that : $(\forall x > 0) \quad f(x) < x$

1 d- Plot the curves (T) and (C) (We take : $\alpha = 0.8$ and $\|\vec{i}\| = \|\vec{j}\| = 2cm$)

Part III : Put $J = \int_0^1 f(x)\,dx$

1 1- a- By using the change of variable $t = \dfrac{1-x}{1+x}$, show that : $J = \dfrac{\pi}{8}\ln 2$

0.5 b- Compute, in cm^2 , the area of the region of the plane enclosed by the curve (C), the tangente (T), the line of equation $x = 0$ and the line of equation $x = 1$

1 2- By using the method of integration by parts calculate : $K = \int_0^1 \dfrac{arc\tan(x)}{1+x}\,dx$

END