الامتحان الوطني الموحد للبكالوريا
المسالك الدولية – خيار أنجليزية
الدورة الاستدراكية 2018
-الموضوع-

RS 24E

المملكة المغربية
وزارة التربية الوطنية
والتكوين المهني
والتعليم العالي والبحث العلمي

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المركز الوطني للتقويم والامتحانات والتوجيه

4	مدة الإنجاز	الرياضيات	المادة
9	المعامل	شعبة العلوم الرياضية : " أ" و " ب" – خيار أنجليزية	الشعبة أو المسلك

- The duration of the exam is 4 hours
- The exam contains 4 independent exercises
- The exercises can be treated in the order chosen by the candidate

- Exercise1 concerns the algebraic structures.....(3.5pts)
- Exercise2 concerns the complex numbers ……..……(3.5pts)
- Exercise3 concerns the calculus of probabilities.....(3pts)
- Exercise4 concerns the analysis……..……..……(10pts)

The use of calculators is not allowed

The use of red pens on answers sheets is not permitted

Exercise 1 : (3.5 pts)

Recall that $(\mathbb{R},+,\times)$ is a commutative field and that $(M_2(\mathbb{R}),+,\times)$ is a real vector space with dimension 4, and that $(M_2(\mathbb{R}),+,\times)$ is a ring with unity $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, its additive identity is the null matrix $O=\begin{pmatrix}0&0\\0&0\end{pmatrix}$.

We put $M(x,y)=\begin{pmatrix}x&y\\0&x\end{pmatrix}$ for all $(x,y)\in\mathbb{R}^2$ and we consider the set

$E=\{M(x,y)/(x,y)\in\mathbb{R}^2\}$

0.5 1- Show that E is a sub-group of the group $(M_2(\mathbb{R}),+)$

0.5 2-a) Show that E is sub-space of the vector space $(M_2(\mathbb{R}),+,.)$

0.25 b) Show that 2 is the dimension of the real vector space $(E,+,.)$

0.25 3- a) Show that E is stable in $(M_2(\mathbb{R}),\times)$

0.5 b) Show that $(E,+,\times)$ is a commutative ring.

4- - We define in $M_2(\mathbb{R})$ the internal composition law T by :

$\forall (x,y,x',y')\in\mathbb{R}^4 \quad M(x,y)\,T\,M(x',y')=M(x,y)\times M(x',y')-M(y,0)\times M(y',0)$

and we consider the mapping φ from $\mathbb{R}^*$ to E defined by

$\forall (x,y)\in\mathbb{R}^2 \quad \varphi(x+iy)=M(x,y)$

0.25 a) Show that E is stable for the law composition T

0.25 b)Show that φ is an homomorphism from $(\mathbb{R}^*,\times)$ to $(E,\times)$

0.25 c) We put $E^*=E-\{O\}$. Show that $(E^*,\times)$ is a commutative group.

0.5 5) a) Show that composition law "T" is distributive on the composition law "+ "

0.25 b- Show that $(E,+,T)$ is a commutative field.

Exercise 2: (3.5 pts)

1) For every complex number $z\in\mathbb{C}-\{i\}$ we put : $h(z)=i\left(\frac{z-2i}{z-i}\right)$

0.5 a) Verify that : $h(z)=z\Leftrightarrow z^2-2iz-2=0$

0.5 b) Solve in $\mathbb{C}$ the equation (E) : $z^2-2iz-2=0$

2- The complex plane is attached to a direct orthonormal coordinate system $(O,\vec{e_1},\vec{e_2})$.

a and b denote the solutions of the equation (E) such that $\mathrm{Re}(a)=1$ and for all $z \in \mathbb{C}-\{i,a,b\}$ $M(z), M'(h(z)), A(a)$ and $B(b)$ denote the points with respective affixes $z, h(z), a$ and b

0.75 a) Show that : $\dfrac{h(z)-a}{h(z)-b} = -\dfrac{z-a}{z-b}$

0.75 b)Deduce that: $\left(\overrightarrow{M'B}, \overrightarrow{M'A}\right) \equiv \pi + \left(\overrightarrow{MB}, \overrightarrow{MA}\right) \ [2\pi]$

0.5 3-a) Show that if the points M, A and B are collinear then the points M, A, B and M' are collinear.

0.5 b) Show that if M, A *and* B are not collinear then M, A, B and M' are cocyclic.

Exercise 3 : (3 pts)

We toss 10 times a fair coin . We consider the random variable equal to the frequency of the apparition of the head.(number of times when head occurs divided by 10)

1 1-a) Determine the values taken by X

1 b) Determine the probability of the event $\left[X = \dfrac{1}{2}\right]$

1 2- Calculate the probability for which X is greater or equal to $\dfrac{9}{10}$

Exercise 4 :(10 pts)

Let f be the numerical function defined on $[0,+\infty[$ by:

$f(x) = \sqrt{x}(\ln x)^2$ for $x>0$ and $f(0)=0$ and Let (C) be the representative curve of f in an orthonormal coordinate system $(O,\vec{i},\vec{j})$

0.5 1-a) Show that f is continuous on right at the point 0

(One can uses : $f(x) = 4x^{\frac{1}{4}} \ln\left(x^{\frac{1}{4}}\right)^2$)

0.75 b) Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty} \dfrac{f(x)}{x}$ and interpret graphically the obtained

result.

0.75 2-a) Study the differentiability on the right of the function f at the point 0 and inerpret graphically the obtained result.

0.75 b) Show that f is differentiable on $]0,+\infty[$ and calculate $f'(x)$ for $x > 0$

1 c) Study the variations of f and deduce that : $(\forall x \in]0,1])$; $0 \le \sqrt{x}(\ln x)^2 \le \left(\frac{4}{e}\right)^2$

0.5 d)Plot the curve (C) (we take 2cm as unit of measur)

3- For all $x \neq 0$ we put : $F(x) = \int_x^1 f(t)\,dt$

0.5 a) Show that the function f is differentiable on the intervall $[0,+\infty[$

1 b) Calculate $F'(x)$ for $x \neq 0$ and then deduce the variations of F on the intervall $[0,+\infty[$

0.75 4-a) By using the method of integration by parts calculate $\int_x^1 \left(\sqrt{t}\ln t\right)dt$ for $x > 0$

0.75 b) Show that for all $x > 0$; $F(x) = -\frac{2}{3}x\sqrt{x}(\ln x)^2 + \frac{8}{9}x\sqrt{x}\ln x - \frac{16}{27}x\sqrt{x} + \frac{16}{27}$

1 c) Deduce in square centimeter (cm^2), the area of the region of the plane enclosed by the curve (C) and the lines with the equations: $x = 0$, $x = 1$ and $y = 0$

5- For all natural number n we put : $u_n = \int_{\frac{1}{n}}^1 f(x)\,dx$

1 a) Show that the sequence $(u_n)_{n\ge 1}$ is bounded and strictly monotonic.

0.75 b) Show that the sequence $(u_n)_{n\ge 1}$ is convergent and calculate $\lim\limits_{n\to+\infty} u_n$

END