# الامتحان الوطني الموحد للبكالوريا

## المسالك الدولية – خيار أنجليزية

الدورة العادية 2018

-الموضوع-

NS 24E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي

المركز الوطني للتقويم والامتحانات والتوجيه

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| المادة | الرياضيات | مدة الإنجاز | 4 |
| الشعبة أو المسلك | شعبة العلوم الرياضية : " أ" و " ب" – خيار أنجليزية | المعامل | 9 |

**- The duration of the exam is 4 hours.**

**- The exam contains 5 independent exercises.**

**- The exercises can be treated in the order chosen by the candidate.**

**- Exercise 1 concerns the algebraic structures.....(3.5pts)**

**- Exercise 2 concerns the arithmetic .....(3 pts)**

**- Exercise 3 concerns the complex numbers .....(3.5pts)**

**- Exercise 4 concerns the analysis.....(7.5pts)**

**- Exercise 5 concerns the analysis.....(2.5pts)**

**The use of calculators is not allowed**

**The use of red pens on answers sheets is not permitted**

**EXERCISE 1** :(3.5 points)

We recall that $(\mathbb{R},+,\times)$ is a commutative field and that $(M_2(\mathbb{R}),+,.)$ is a real vector space and that $(M_2(\mathbb{R}),+,\times)$ is a commutative ring with unity $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, its additive identity is the null matrix $O=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

For every $(x,y)\in\mathbb{R}^2$ we put $M(x,y)=\begin{pmatrix}x & -2y\\ y & x+2y\end{pmatrix}$ and we consider the set $E=\{M(x,y)/(x,y)\in\mathbb{R}^2\}$

0.25 1- Show that $E$ is a sub-group of the group $(M_2(\mathbb{R}),+)$

0.25 2- a) Show that $E$ is a sub-space of the real vector space $(M_2(\mathbb{R}),+,.)$

0.5 b) We put $J=M(0,1)$. Show that $(I,J)$ is a basis of the real vector space $(E,+,.)$

0.5 3-a) Show that $E$ is stable in $(M_2(\mathbb{R}),\times)$

0.5 b) Show that $(E,+,\times)$ is a commutative ring.

4- Let $\varphi$ be the mapping from $\mathbb{C}^*$ to $M_2(\mathbb{R})$ defined by :

$$\left(\forall(x,y)\in\mathbb{R}^2-\{(0,0)\}\right)\quad;\quad \varphi(x+iy)=M(x+y,-y)=\begin{pmatrix}x+y & 2y\\ -y & x-y\end{pmatrix}$$

0.5 a) Show that $\varphi$ is an homomorphism from $(\mathbb{C}^*,\times)$ to $(M_2(\mathbb{R}),\times)$

0.5 b) We put $E^*=E-\{O\}$. Show that : $\varphi(\mathbb{C}^*)=E^*$

0.25 c) Deduce that $(E^*,\times)$ is a commutative group.

0.25 5- Show that $(E,+,\times)$ is a commutative field .

**EXERCISE 2**: (3 points)

Let $p$ be a prime number such that $p=3+4k$ $(k\in\mathbb{N}^*)$

0.5 1- Show that for all integer number $x$, if $x^2\equiv 1\ [p]$ then $x^{p-5}\equiv 1\ [p]$

2- Let $x$ be an integer number verifying : $x^{p-5}\equiv 1\ [p]$

0.5 a) Show that $x$ and $p$ are coprime.

0.5 b) Show that : $x^{p-1} \equiv 1 \ [p]$

0.5 c) Verify that : $2+(k-1)(p-1)=k(p-5)$

0.5 d) Deduce that : $x^2 \equiv 1 \ [p]$

0.5 3 ) Solve in $\mathbb{Z}$ the equation : $x^{62} \equiv 1 \ [67]$

**EXERCISE 3** : (3.5 points)

Let $m$ be a complex number.

I- We consider in the set of the complex numbers $\mathbb{C}$ the equation $(E_m)$ with variable $z$ :

$$z^2+(im+2)z+im+2-m=0$$

0.25 1-a) Verify that $\Delta=(im-2i)^2$ is the discriminant of the equation $(E_m)$

0.5 b) Give according the values of $m$ , the set of the solutions of the equation $(E_m)$

0.5 2- For $m=i\sqrt{2}$ , rewrite the two solutions of the equation $(E_m)$ in exponential polar form.

II- The complex plane is attached to a direct orthonormal coordinate system $(O;\vec{u},\vec{v})$

We consider the points $A$ , $\Omega$ , $M$ and $M'$, with respective affixes $a=-1-i$ , $\omega=i$ , $m$ and $m'=-im-1+i$

1- Let $R$ be the rotation with angle $-\frac{\pi}{2}$ which send $M$ to $M'$

0.25 a) Verify that $\Omega$ is the center of the rotation $R$

0.5 b) Determine the affix $b$ of $B$, where $B$ is the point verifying : $A=R(B)$

0.5 2-a) Verify that : $m'-a=\frac{\omega-a}{\omega-b}(m-b)$

0.5 b) Deduce that the points $A$ , $M$ and $M'$ are collinear if and only if the points $A$ , $B$, $\Omega$ and $M$ are cocyclic.

0.5 c) Show that the set of the points $M$ such that the points $A$ , $M$ and $M'$ are collinear is a circle which center and radius are to be determined.

**EXERCISE 4** :(7.5 points)

**PART I**

0.5 1-a) Show that : $(\forall x \in ]0,+\infty[)$ ; $\int_0^x \frac{t}{1+t}dt = x-\ln(1+x)$

b) By using the change of variable $u=t^2$ , show that :

0.5 $(\forall x \in ]0,+\infty[)$; $\int_0^x \frac{t}{1+t}dt = \frac{1}{2}\int_0^{x^2} \frac{1}{1+\sqrt{u}}du$

0.5 c) Deduce that : $\left(\forall x\in ]0,+\infty[\right)$ ; $\dfrac{1}{2(1+x)}\le\dfrac{x-\ln(1+x)}{x^2}\le\dfrac{1}{2}$

0.25 2- Determine : $\lim\limits_{x\to 0^+}\dfrac{x-\ln(1+x)}{x^2}$

**PART II**

We consider the numerical function $f$ defined on $[0,+\infty[$ by :

$$\begin{cases} f(x)=\left(\dfrac{x+1}{x}\right)\ln(1+x) \;\; ; \;\; x\neq 0 \\ f(0)=1 \end{cases}$$

and let $(C)$ be the representative curve of $f$ in an orthonormal coordinate system $\left(O;\vec{i},\vec{j}\right)$

0.25 1-a)Show that $f$ is right-continuous at the point $0$

b) Show that $f$ is differentiable on the right at the point 0

0.5 (One can use the result of the question I-2)

0.75 c) Compute : $\lim\limits_{x\to+\infty} f(x)$ , $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}$ and interpret graphically the obtained result.

0.5 2- a) Show that $f$ is differentiable on $]0,+\infty[$ , and verify that :

$$\left(\forall x\in ]0,+\infty[\right) \;\; ; \;\; f'(x)=\frac{x-\ln(1+x)}{x^2}$$

0.25 b) Deduce that $f$ is strictly increasing on $[0,+\infty[$

0.25 c) Verify that : $f\left([0,+\infty[\right)=[1,+\infty[$

0.5 3- Plot the curve $(C)$ (the right half-tangent at the point 0 must be ploted )

**PART III**

1- Let $g$ be the numerical function defined on $]0,+\infty[$ by: $g(x)=f(x)-x$

0.5 a) Show that : $\left(\forall x\in ]0,+\infty[\right)$ ; $0\le f'(x)\le\dfrac{1}{2}$

0.5 b) Deduce that $g$ is strictly decreasing on $]0,+\infty[$ and show that $g\left(]0,+\infty[\right)=]-\infty,1[$

0.25 c) Show that the equation $f(x)=x$ admits a unique solution $\alpha$ in $]0,+\infty[$

2- Let $a$ be a real number in $]0,+\infty[$ .

Consider the sequence $(u_n)_{n\ge 0}$ defined by : $u_0=a$ and $\left(\forall n\in\mathbb{N}\right)$ ; $u_{n+1}=f(u_n)$

0.25 a) Demonstrate that : $\left(\forall n\in\mathbb{N}\right)$ ; $u_n>0$

0.5 b) Show that : $\left(\forall n\in\mathbb{N}\right)$ ; $|u_{n+1}-\alpha|\le\dfrac{1}{2}|u_n-\alpha|$

0.5 c) Prove by induction that : $(\forall n \in \mathbb{N})$ ; $|u_n - \alpha| \leq \left(\frac{1}{2}\right)^n |a - \alpha|$

0.25 d) Show that the sequence $(u_n)_{n\geq 0}$ converge to $\alpha$

**EXERCISE 5** : (2.5 points)

Consider the numerical function $F$ defined on $\mathbb{R}$ by: $F(x) = \int_0^x e^{t^2} dt$

0.5 1- Show that $F$ is continuous and is strictly increasing on $\mathbb{R}$

0.5 2-a) Show that : $(\forall x \in ]0, +\infty[)$ ; $F(x) \geq x$ , and then deduce $\lim\limits_{x \to +\infty} F(x)$

0.5 b) Show that $F$ is odd , and then deduce $\lim\limits_{x \to -\infty} F(x)$

0.5 c) Show that $F$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$

0.5 d) Show that the inverse bijection $G$ of the function $F$ is differentiable at 0 and compute $G'(0)$

**END**