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المسالك الدولية ـ خيار إنجليزية
الدورة الاستدراكية 2017
- الموضوع -

RS 24E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي

المركز الوطني للتقويم والامتحانات والتوجيه

المادة	الرياضيات	مدة الإنجاز	4
الشعبة أو المسلك	مسلك العلوم الرياضية (أ) و(ب) – خيار إنجليزية	المعامل	9

- The duration of the exam is 4 hours
- The exam contains 4 independent exercises
- The exercises can be treated in the order chosen by the candidate

- Exercise1 concerns the algebraic structures.....(4.5pts)
- Exercise2 concerns the calculus of probabilities.....(3pts)
- Exercise3 concerns the complex numbers .....(2.5pts)
- Exercise4 concerns the analysis.....(10pts)

The use of calculators is not allowed
The use of red pens on answers sheets is not permitted

Exercise 1 : (4.5 pts)

Recall that $(\mathbb{R},+,\times)$ is a commutative field and that $(M_2(\mathbb{R}),+,\cdot)$ is a real vector space and that $(M_2(\mathbb{R}),+,\times)$ is a unitary ring which is not integral and not commutative.

We put : $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $J=\begin{pmatrix}0&-3\\1&0\end{pmatrix}$ and $M(x,y)=\begin{pmatrix}x&-3y\\y&x\end{pmatrix}$ for all $(x,y)\in\mathbb{R}^2$

and $E=\{M(x,y)/(x,y)\hat{I}\ ¡^2\}$

0.75 1- Show that E is a sub-space of $(M_2(\mathbb{R}),+,\cdot)$ with dimension 2

0.5 2-a) Show that E is stable in $(M_2(\mathbb{R}),\times)$

0.75 b) Show that $(\mathrm{E},+,\times)$ is a unitary commutative ring.

3- We put $E^*=E\setminus\{M(0,0)\}$ and we consider the mapping φ from $\mathbb{R}^*$ to E^* defined by $\forall(x,y)\in\mathbb{R}^2$ $\varphi(x+iy)=M\left(x,\frac{y}{\sqrt{3}}\right)$

0.75 a) Show that φ is an isomorphism from $(\mathbb{R}^*,\times)$ to $(E^*,\times)$

0.5 b) Deduce that $(\mathrm{E}^*,\times)$ is a commutative group.

0.75 c) Show that : $J^{2017}=\mathrm{j}\left(3^{1008}\sqrt{3}i\right)$, and determine the inverse of the matrix J^{2017} in $(\mathrm{E}^*,\times)$

0.5 4- Show that $(\mathrm{E},+,\times)$ is a commutative field.

Exercise 2 : (3 pts)

A bag contains 2n balls ($\mathrm{n}\hat{I}\ ¥^*$), n balls are white and n balls are black. All the balls are indistinguishable by the touch.

A game consists of : to draw randomly one ball from the bag, note its color and replace it in the bag, then to draw randomly one ball from the same bag and note its color.

The rule of the game indicates that:

- If the two drawn balls are white we win 20 points.
- if the two drawn balls are black we lose 20 points.
- If the drawn balls have different color the gain is O points.

0.75 1- Calculate the probability to win 20 points, the probability to lose 20 points and the probability to realize a zero points.

2- We repeat the previous game 5 times.

0.5 a) Calculate the probability to win 100 points.

1 b) Calculate the probability to win 40 points.

3- During the initial game, consider the random variable X which takes - 20 if we lose, 0 if the gain is O points and +20 if we win.

0.5 a) Determine the law of probability of the random variable X.

0.25 b) Calculate the expected value of the random variable X

Exercise 3: (2.5 pts)

The complex plane is attached to a direct orthonormal coordinate system $\left(O,\vec{e}_1,\vec{e}_2\right)$

Let M be the point with affix a non zero complex number z and the point M' with affix $z'=\frac{1}{2}\left(z+\frac{1}{z}\right)$

0.5 1- Determine the complex number z for which M and M' are identical

2- We assume that M is distinct from the points A and B which affixes are respectively 1 and -1.

0.5 Show that : $\frac{z'+1}{z'-1}=\left(\frac{z+1}{z-1}\right)^2$

0.75 3- (Δ) is the perpendicular bisector of the segment $[AB]$

Show that: if M belongs to (Δ) then M' also belongs to (Δ)

4- (Γ) is the circle with diameter $[AB]$

0.75 Show that : if M belongs to (Γ) then M' belongs to the line (AB)

Exercise 4 :(10 pts)

Part1 :

Let f be the numerical function defined on $I=[0,+\infty[$ by:

$f(0)=1$ and $\forall x\in]0,+\infty[$; $f(x)=\frac{\arctan(x)}{x}$

0.5 1- Show that f is continuous on the interval I

0.5 2-a) Let x in I. Show that : $\forall t\in[0,x]$; $\frac{1}{1+x^2}\le\frac{1}{1+t^2}\le 1$

0.5 b) Show that : $\forall x\in[0,+\infty[$; $\frac{x}{1+x^2}\le\arctan(x)\le x$

0.75 c) Show that f is differentiable on the right at the point 0

0.5 3- a) Knowing that f is differentiable on the interval $]0,+\infty[$, calculate $f'(x)$ for x in $]0,+\infty[$

0.25 b) Study the variation of f on interval I

Part 2 :

Let g be the function defined on $I=[0,+\infty[$ by :

$g(0)=1$ and $\forall x\in]0,+\infty[\quad g(x)=\frac{1}{x}\int_0^x f(t)dt$

0.5 1- a) Show that: $\forall x\in]0,+\infty[$; $f(x)\le g(x)\le 1$

0.75 b) Show that g is differentiable on the right at the point 0

0.75 2- Show that the function g is differentiable on the interval $]0,+\infty[$ and :

$\forall x\in]0,+\infty[\quad g'(x)=\frac{1}{x}\left(f(x)-g(x)\right)$

0.25 3- Show that g is decreasing on I

0.75 4-a) Show that : $\lim\limits_{x\to+\infty}\frac{1}{x}\int_1^x f(t)dt=0$

(note that : $\forall x\in]0,+\infty[\quad 0<\arctan x<\frac{\pi}{2}$)

0.5 b) Calculate : $\lim\limits_{x\to+\infty} g(x)$

Part 3 :

0.75 1- Show that the equation $g(x)=x$ has a single solution α in $]0,1[$

0.5 2- a) Verify that : $\forall x\in [0,+\infty[\quad 0\le 1-f(x)\le \frac{x^2}{1+x^2}$

(one can use the question 2-a)Part1)

0.75 b) Show that : $\forall x\in [0,+\infty[\quad |g'(x)|\le \frac{1}{2}$

3- Let $(u_n)_{n\ge 0}$ be the sequence defined by : $u_0\in \mathbb{R}^+$, $u_{n+1}=g(u_n)$ for all n in $\mathbb{N}$

0.75 a) Show that : $\forall n\in\mathbb{N} \quad |u_{n+1}-\alpha|\le\frac{1}{2}|u_n-\alpha|$

0.75 b) Show that the sequence $(u_n)_{n\ge 0}$ is convergent.

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