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الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة الاستدراكية 2022
– الموضوع –

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المملكة المغربية
وزارة التربية الوطنية والتعليم الأولي والرياضة
المركز الوطني للتقويم والامتحانات

3h	مدة الإنجاز	الرياضيات	المادة
7	المعامل	مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية - خيار إنجليزية	الشعبة أو المسلك

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

- ✓ **The exam consists of four exercises and a problem , independent of each other according to the fields as follows :**

Exercise 1	**numerical sequences**	**2,5 points**
Exercise2	**Geometry in space**	**3 points**
Exercise 3	**Complex numbers**	**3 points**
Exercise 4	**Calculating probabilities**	**3 points**
Problem	**Study of numerical function and calculating integrals**	**8,5 points**

- ✓ **ln denotes the Napierian logarithm function**

Exercise 1 (*2,5* points) :

Let (u_n) be the numerical sequence defined by $u_0 = 2$ and $u_{n+1} = \frac{\sqrt{2}}{2}u_n + \frac{2-\sqrt{2}}{2}$ for every n on IN

0,5 **1) a) Show that for every n on IN, $u_n > 1$**

0,75 **b) Show that for every n on IN, $u_{n+1} - u_n = \frac{\sqrt{2}-2}{2}(u_n - 1)$ and deduce that the sequence (u_n) is decreasing and convergent.**

2) Let $v_n = u_n - 1$, for every n on IN

0,5 **a) Show that (v_n) is a geometrical sequence and determine its ratio and its first term.**

0,5 **b) Write u_n in term of n then deduce the limit of the sequence (u_n).**

0,25 **c) Calculate the sum $S = u_0 + u_1 + u_2 + ... + u_{2021}$**

Exercise 2 (3 points) :

In the space referred to an orthonormal direct coordinate system $(O, \vec{i}, \vec{j}, \vec{k})$, we consider the two points $A(1,-1,1)$ and $B(5,1,-3)$. Let (S) be the sphere of center $\Omega(3,0,-1)$ and the radius $R = 3$, and (Δ) be the line passing through the point A and with direction vector $\vec{u}(2,-2,1)$,

0,25 **1) a) Calculate the distance ΩA**

0,5 **b) Show that the lines (Δ) and (ΩA) are perpendicular.**

0,25 **c) Deduce the position of the line (Δ) and the sphere (S)**

2) Let $a \in \mathbb{R}$ and M_a be the point with coordinates $M_a(2a-3, 3-2a, a-1)$,

0,5 **Show that $\overrightarrow{AM_a} = (a-2)\vec{u}$ and deduce that $M_a \in (\Delta)$ for every $a \in \mathbb{R}$**

0,5 **3) a) Verify that $2x - 2y + z - 9a + 13 = 0$ is an equation of the plane (P_a) passing through M_a and perpendicular to the line (Δ)**

0,5 **b) Show that $d(\Omega, (P_a)) = |3a - 6|$**

0,5 **c) Determine the two values of a for which the plane (P_a) is tangent to the sphere (S).**

Exercise 3 (3 points) :

In the complex plane referred to an orthonormal direct coordinate system $(O; \vec{u}, \vec{v})$, We consider the points A, B and C of respective affixes $Z_A = 1+5i$, $Z_B = 1-5i$ and $Z_C = 5-3i$

0,25 **1) Determine the complex number Z_D the affix of D midpoint of the segment $[AC]$**

0,5 **2) let h be the homothety of the center A and the ratio $\frac{1}{2}$;**

Determine the complex number Z_E affix of the point E image of B by h

0,5 **3) We consider the rotation R of center C and angle $\left(\frac{-\pi}{2}\right)$, determine the image of B by R**

4) Let F be the point of affix $Z_F = -1+i$

0,25 **a) Verify that $\frac{Z_D - Z_A}{Z_F - Z_A} \times \frac{Z_F - Z_E}{Z_D - Z_E} = -1$**

0,5 **b) Deduce that $\left(\overrightarrow{AF}, \overrightarrow{AD}\right) + \left(\overrightarrow{ED}, \overrightarrow{EF}\right) \equiv \pi [2\pi]$**

0,5 **c) Determine the trigonometric form of the number $\frac{Z_E - Z_F}{Z_A - Z_F}$, and deduce the nature of the triangle AEF**

0,5 **d) Deduce that the points A, D, E and F are on a circle with a diameter will be determined.**

Exercise 4 (3 points) :

An urn contains three white balls, four red balls and five green balls, indistinguishable by touch. We draw randomly and simultaneously three balls from the urn.

1) Consider the following events: A **« Obtaining exactly two red balls »**
B **« Obtaining exactly one green ball »**

0,75 **a) Show that $p(A) = \frac{12}{55}$ and $p(B) = \frac{21}{44}$**

0,75 **b) Calculate $p(A/B)$: the probability of event A knowing that event B is realized.**

Are the events A and B independents?

2) We consider the random variable X which associates to each draw the number of green balls drawn.

1 **a) Determine the probability law of the random variable X .**

0,5 **b) Calculate the probability to obtain at least two green balls.**

Problem (8,5 points) :

Let f be the numerical function defined on $[0, +\infty[$ by $\begin{cases} f(x) = x^4(\ln x - 1)^2 \ ; \ x > 0 \\ f(0) = 0 \end{cases}$

and (C) its representative curve in an orthonormal coordinate system $(O; \vec{i}, \vec{j})$ (unit: $1cm$)

0,75 **1) Calculate $\lim\limits_{x \to +\infty} f(x)$ then determine the infinite branch of (C) at $+\infty$**

0,5 **2) a) Show that f is continuous on the right at 0**

0,5	**b) Study the differentiability of the function f on the right at 0 and interpret geometrically the result**
0,75	**3) a) Show that $f'(x)=2x^3(\ln x-1)(2\ln x-1)$ for every x on the interval $]0,+\infty[$**
0,5	**b) Set up the table of variations of the function f**
0,5	**4) a) Knowing that $f''(x)=2x^2(6\ln x-5)\ln x$ for every x on the interval $]0,+\infty[$, study the sign of $f''(x)$ on $]0,+\infty[$**
0,5	**b) Deduce that the curve (C) admits two points of inflection which the abscissae will be determined**
1	**5) a) Sketch the curve (C) on the coordinate system $\left(O,\vec{i},\vec{j}\right)$ (take: $\sqrt{e}\approx 1,6$ and $e^2\approx 7,2$)**
0,5	**b) Using the curve (C), determine the number of solutions of the equation $x^2(\ln x-1)=-1$**
	6) We consider the function g defined on $\mathbb{R}$ by $g(x)=f\left(\lvert x\rvert\right)$
0,5	**a) Show that g is an even function.**
0,5	**b) Sketch (C_g) the curve of g on the same coordinate system $\left(O,\vec{i},\vec{j}\right)$**
0,5	**7) a) Consider $I=\int_1^e x^4(\ln x-1)\,dx$, using an integration by parts, Show that $I=\frac{6-e^5}{25}$**
0,5	**b) We consider the function h defined on the interval $]0,+\infty[$ by $h(x)=x^5(\ln x-1)^2$.** **Verify that $h'(x)=5f(x)+2x^4(\ln x-1)$**
0,5	**c) Deduce that $\int_1^e f(x)\,dx=-\frac{1}{5}-\frac{2}{5}I$**
0,5	**d) Calculate the area of the domain delimited by the curve (C) and x-axis and the lines of equations $x=1$ and $x=e$**