الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة العادية 2022
– الموضوع –

المملكة المغربية
وزارة التربية الوطنية والتعليم الأولي والرياضة
المركز الوطني للتقويم والامتحانات

SSSSSSSSSSSSSSSSSS-SS | NS 22E

المادة	الرياضيات	مدة الإنجاز	3h
الشعبة أو المسلك	مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية - خيار إنجليزية	المعامل	7

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

- ✓ **The exam consists of four exercises and a problem , independent of each other according to the fields as follows :**

Exercise 1	**Geometry in space**	**3 points**
Exercise 2	**Complex numbers**	**3 points**
Exercise 3	**Calculating probabilities**	**3 points**
Exercise 4	**Differential equations and calculating integrals**	**2.5 points**
Problem	**Study of numerical functions and numerical sequences**	**8.5 points**

- ✓ $\bar{z}$ **denotes the conjugate of the complex number** z **, and** $|z|$ **its modulus**
- ✓ **ln denotes the Napierian logarithm function**

Exercise 1 (*3 points*) :

In the space referred to a direct orthonormal coordinate system $(O,\vec{i},\vec{j},\vec{k})$ we consider the points $A(0,1,1)$, $B(1,2,0)$ and $C(-1,1,2)$

0,5 1) a) Show that $\overrightarrow{AB}\wedge\overrightarrow{AC}=\vec{i}+\vec{k}$

0,25 b) Deduce that $x+z-1=0$ is a cartesian equation of the plane (ABC)

0,5 2) Let (S) be the sphere with the center $\Omega(1,1,2)$ and the radius $R=\sqrt{2}$. Determine an equation of the sphere (S)

0,5 3) Show that the plane (ABC) is tangent to the sphere (S) at the point A

4) We consider the line (Δ) passing through the point C and perpendicular to the plane (ABC)

0,25 a) Determine a parametric equation of the line (Δ)

0,5 b) Show that the line (Δ) is tangent to the sphere (S) at a point D whose the coordinates will be determined.

0,5 c) Calculate the scalar product $\overrightarrow{AC}.(\vec{i}+\vec{k})$ then deduce the distance $d(A,(\Delta))$.

Exercise 2 (*3 points*) :

In the complex plane referred to a direct orthonormal coordinate system $(O,\vec{u},\vec{v})$, we consider the point A of affix $a=-1-i\sqrt{3}$, the point B of affix $b=-1+i\sqrt{3}$ and the translation t of vector $\overrightarrow{OA}$

0,5 1) Prove that the affix of the point D image of the point B by the translation t is $d=-2$

2) We consider the rotation R with center D and angle $\left(\frac{2\pi}{3}\right)$.

0,5 Show that the affix of the point C image of the point B by the rotation R is $c=-4$

0,5 3) a) Write the number $\frac{b-c}{a-c}$ in the trigonometric form

0,5 b) Deduce that $\left(\frac{b-c}{a-c}\right)^2=\frac{c-d}{b-d}$

4) Let (Γ) be the circle with center D and radius 2, (Γ') the circle with center O and radius 4 and M a point of affix z belonging to the two circles (Γ) and (Γ')

0,25 a) Verify that $|z+2|=2$

0,5 b) Prove that $z+\overline{z}=-8$ (notice that $|z|=4$)

0,25 c) Deduce that the circle (Γ) intersects (Γ') at an unique point which will be determined.

Exercise 3 (*3 points*) :

An urn contains ten balls: three white balls, three green balls and four red balls indistinguishable by touch. We randomly draw simultaneously three balls from the urn.

0,75 1) Show that $p(A)=\frac{1}{6}$; where A is the event " Not getting any red ball "

0,75 2) Calculate $p(B)$; where B is the event " Getting three white balls or three green balls "

0,75 3) Show that $p(C)=\frac{1}{2}$; where C is the event " Getting exactly one red ball "

0,75 4) Calculate $p(D)$; where D is the event " Getting at least two red balls "

Exercise 4 (*2.5 points*) :

We consider the function h defined on $\mathbb{R}$ by $h(x)=(x+1)\mathrm{e}^{x}$

0,75 1) a) verify that the function $x\to xe^{x}$ is a primitive of h on $\mathbb{R}$ then calculate $I=\int_{-1}^{0}h(x)\,dx$

0.75 b) Using an integration by parts; calculate $J=\int_{-1}^{0}(x+1)^2e^{x}\,dx$

0,5 2) a) Solve the differential equation $(E): y''-2y'+y=0$

0,5 b) Show that the function h is the solution of the equation (E) which satisfies the conditions $h(0)=1$ and $h'(0)=2$

Problem (*8.5 points*) :

We consider the numerical function f defined on $\mathbb{R}$ by $f(x)=x(\mathrm{e}^{\frac{x}{2}}-1)^2$.

Let (C) be the curve of f in an orthonormal coordinate system $(O;\vec{i},\vec{j})$ (unit : $1\,cm$)

0,5 1) Calculate $\lim\limits_{x\to+\infty}f(x)$ and $\lim\limits_{x\to-\infty}f(x)$

0,5 2) Calculate $\lim\limits_{x\to+\infty}\frac{f(x)}{x}$ and interpret geometrically the result .

0,5 3) a) Show that the line (Δ) with equation $y=x$ is an asymptote to the curve (C) at $-\infty$

0,75 b) Study the sign of $(f(x)-x)$ for all x in $\mathbb{R}$ and deduce the relative position of the curve (C) and the line (Δ)

0,5 4) a) Show that $f'(x)=(\mathrm{e}^{\frac{x}{2}}-1)^2+x\mathrm{e}^{\frac{x}{2}}(\mathrm{e}^{\frac{x}{2}}-1)$ for all x on $\mathbb{R}$

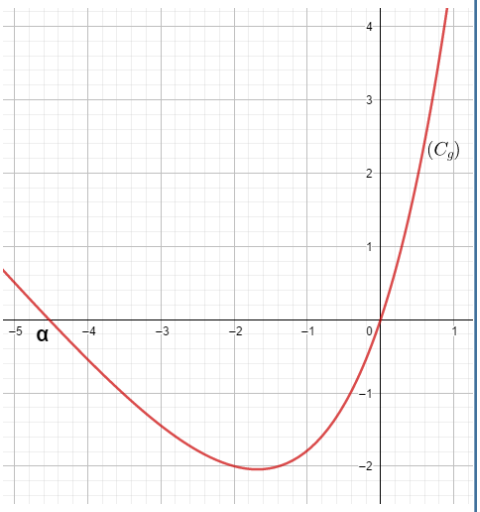
0,5 b) Verify that $x(\mathrm{e}^{\frac{x}{2}}-1)\geq 0$ for all x in $\mathbb{R}$ then deduce the sign of the derived function f' on $\mathbb{R}$

0,25 c) Set up the table of variations of the function f in $\mathbb{R}$

0,5 5) a) Show that $f''(x)=\frac{1}{2}e^{\frac{x}{2}}g(x)$; where $g(x)=(2x+4)e^{\frac{x}{2}}-x-4$ for all x in $\mathbb{R}$

0,5 b) From the opposite curve of the function g,

Determine the sign of $g(x)$ on $\mathbb{R}$ (Notice : $g(\alpha)=0$)



0,5 c) Study the concavity of the curve (C), and determine the abscissas of the two inflection points.

1 6) Sketch the curve (C) in the coordinate system $(O;\vec{i},\vec{j})$

(We take : $\ln(4)\approx 1,4$, $\alpha\approx -4,5$ and $f(\alpha)\approx -3,5$)

0,5 7) a) Show that the function f admits an inverse function f^{-1} defined on $\mathbb{R}$

0,25 b) Calculate $\left(f^{-1}\right)'(\ln 4)$

8) Let (u_n) be the numerical sequence defined by $u_0=1$ and $u_{n+1}=f(u_n)$ for every n in $\mathbb{N}$

0,5 a) Show by induction that $0<u_n<\ln 4$ for every n in $\mathbb{N}$

0,5 b) Show that the sequence (u_n) is decreasing.

0,25 c) Deduce that the sequence (u_n) is convergent.

0,5 d) Calculate the limit of the sequence (u_n).