الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة العادية 2021
- الموضوع –

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

SSSSSSSSSSSSSSSSSS | NS 22E

3h	مدة الإنجاز	الرياضيات	المادة
7	المعامل	شعبة العلوم التجريبية مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية (خيار إنجليزية)	الشعبة أو المسلك

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

- ✓ **The exam consists of three exercises and a problem , independent of each other according to the fields as follows :**

Exercise 1	**numerical functions**	**2 points**
Exercise 2	**numerical sequences**	**4 points**
Exercise 3	**Complex numbers**	**5 points**
Problem	**Study of numerical functions and calculating integrals**	**9 points**

- ✓ **$\overline{z}$ denotes the conjugate of the complex number z**
- ✓ **ln denotes the Napierian logarithm function**

Exercise 1 : (2 points)

0.5 1) a) Solve in $\mathbb{R}$ the equation with variable x : $e^{2x}-4e^{x}+3=0$

0.5 b) Solve in $\mathbb{R}$ the inequation with variable x : $e^{2x}-4e^{x}+3\leq 0$

0.5 c) Calculate $\lim\limits_{x\to 0}\dfrac{e^{2x}-4e^{x}+3}{e^{2x}-1}$

0.5 2) Show that the equation $e^{2x}+e^{x}+4x=0$ admits a solution in the interval $[-1,0]$

Exercise 2 : (4 points)

Let (u_n) be the numerical sequence defined by $u_0=\dfrac{1}{2}$ and $u_{n+1}=\dfrac{u_n}{3-2u_n}$ for every natural number n

0.25 1) Calculate u_1

0.5 2) Show by induction that $0<u_n\leq\dfrac{1}{2}$ for every natural number n

0.5 3) a) Show that $\dfrac{u_{n+1}}{u_n}\leq\dfrac{1}{2}$ for every natural number n

0.5 b) Deduce the monotonicity of the sequence (u_n)

0.75 4) a) Show that for every natural number n, $0<u_n\leq\left(\dfrac{1}{2}\right)^{n+1}$, then calculate $\lim u_n$

0.5 b) We put $v_n=\ln(3-2u_n)$ for every natural number n, calculate $\lim v_n$

0.5 5) a) Verify that for every natural number n, $\dfrac{1}{u_{n+1}}-1=3\left(\dfrac{1}{u_n}-1\right)$

0.5 b) Deduce u_n in term of n for every natural number n.

Exercise 3 : (5 points)

0.75 1) Solve in the set of complex numbers $\mathbb{C}$ the equation $z^2-\sqrt{3}z+1=0$

2) Let be the complex numbers $a=e^{i\frac{\pi}{6}}$ and $b=\dfrac{3}{2}+i\dfrac{\sqrt{3}}{2}$

0.25 a) Write a in the algebraic form

0.5 b) Verify that $\overline{a}b=\sqrt{3}$

In the complex plane referred to a direct orthonormal coordinate system $(O,\vec{u},\vec{v})$, we consider the points A,B and C of respective affixes a,b and $\overline{a}$.

0.5 3) Show that the point B is the image of the point A by the homothety h with center O and whose ratio will be determined.

4) Let z be the affix of a point M in the plane and z' the affix of the point M', image of M by the rotation R with center A and angle $\frac{\pi}{2}$

0.5 a) Write z' in term of z and a.

0.25 b) Let d be the affix of a point D, image of C by the rotation R, show that $d = a+1$

0.5 c) Let I be the point of affix the number 1, show that $ADIO$ is a rhombus.

0.75 5) a) Verify that $d-b=\frac{\sqrt{3}-1}{2}(1-i)$ and deduce an argument of the number $d-b$

0.5 b) Write the number $1-b$ in trigonometrical form.

0.5 c) Deduce a measure of the angle $\left(\overrightarrow{BI};\overrightarrow{BD}\right)$

Problem : (9 points)

Consider the numerical function f defined on $[0,+\infty[$ by:
$f(0)=0$ and $f(x)=2x\ln x-2x$ if $x>0$ and let (C) be its representative curve in an orthonormal coordinate system $\left(O,\vec{i},\vec{j}\right)$ (unit: $1cm$)

0.5 1) Show that f is continuous at the point 0 on the right.

0.5 2) a) Calculate $\lim\limits_{x\to+\infty} f(x)$

0.5 b) Calculate $\lim\limits_{x\to+\infty}\frac{f(x)}{x}$ then interpret geometrically the result.

0.75 3) a) Calculate $\lim\limits_{x\to0^+}\frac{f(x)}{x}$ then interpret geometrically the result.

0.5 b) Calculate $f'(x)$ for every x on $]0,+\infty[$

0.5 c) Set up the table of variations of the function f on $[0,+\infty[$

0.5 4) a) Solve in the interval $]0,+\infty[$ the equations $f(x)=0$ and $f(x)=x$

1 b) Sketch the curve (C) in the coordinate system $\left(O,\vec{i},\vec{j}\right)$ (we take $e^{\frac{3}{2}}\simeq 4.5$)

0.5 5) a) Using an integration by parts, show that $\int_1^e x\ln x\,dx=\frac{1+e^2}{4}$

0.5 b) Deduce $\int_1^e f(x)\,dx$

0.25 6) a) Determine the minimum of f on $]0,+\infty[$

0.5 b) Deduce that for every x on $]0,+\infty[$, $\ln x\geq\frac{x-1}{x}$

7) Let g be the restriction of the function f on the interval $[1,+\infty[$

0.5 a) Show that g admits an inverse function g^{-1} defined on an interval J which will be determined.

0.75 b) Sketch in the same coordinate system $(O,\vec{i},\vec{j})$ the curve of the function g^{-1}

8) Consider the function h defined on $\mathbb{R}$ by $\begin{cases} h(x)=x^3+3x & ; \ x\leq 0 \\ h(x)=2x\ln x-2x & ; \ x>0 \end{cases}$

0.5 a) Study the continuity of h at the point 0

0.5 b) Study the differentiability of h at the point 0 on the left, then interpret geometrically the result.

0.25 c) The function h is-it differentiable at the point 0 ? justify.