الامتحان الوطني الموحد للبكالوريا
المسالك الدولية – خيار أنجليزية
الدورة الاستدراكية 2018
-الموضوع-

RS 22E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي

المركز الوطني للتقويم والامتحانات والتوجيه

المادة	الرياضيات	مدة الإنجاز	3
الشعبة أو المسلك	مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية – خيار أنجليزية	المعامل	7

GENERAL INSTRUCTIONS

- ✓ **The use of non- programmable calculator is allowed ;**
- ✓ **The exercises can be treated in the preferred order by the candidate ;**
- ✓ **The use of red color when writing solutions is to be avoided.**

COMPONENTS OF THE EXAM

✓ **The exam consists of four exercises and a problem , independent of each other according to the fields as follows :**

Exercise 1	**Geometry in space**	**3 points**
Exercise 2	**Complex numbers**	**3 points**
Exercise 3	**Calculating probabilities**	**3 points**
Exercise 4	**calculating integrals**	**2 points**
Problem	**Study of numerical function, and numerical sequences**	**9 points**

- ✓ **ln denotes the Napierian logarithm function**

Exercise 1: (3 points)

In the space referred to an orthonormal direct coordinate system $(O,\vec{i},\vec{j},\vec{k})$, we consider the sphere (S) with the center $\Omega(2,1,2)$ and the radius 3, and the plane (P) passing through the point $A(-1,\ 0,\ 3)$ and having $\vec{u}(4,\ 0,\ -3)$ as a normal vector.

0.5 1) Show that an equation of the sphere (S) is $x^2+y^2+z^2-4x-2y-4z=0$

0.5 2) Verify that a cartesian equation of the plane (P) is $4x-3z+13=0$

0.5 3) a) Verify that $\begin{cases} x=2+4t \\ y=1 \\ z=2-3t \end{cases}$ $(t\in\mathbb{R})$ is a parametric equations of the line (Δ) passing through Ω and perpendicular to the plane (P)

0.5 b) Determine the coordinates of H, the point of intersection of the line (Δ) and the plane (P)

0.25 4) a) Calculate $d(\Omega,(P))$

0.75 b) Show that the plane (P) is tangent to the sphere (S) at a point wich will be determined.

Exercise 2 : (3 points)

0.75 1) Solve in the set of complex numbers $\mathbb{C}$ the equation $z^2-2\sqrt{2}z+4=0$

2) In the complex plane referred to an orthonormal direct coordinate system $(O,\vec{u},\vec{v})$, we consider the point A of affix $a=\sqrt{2}(1-i)$ and the rotation R with center O and angle $\frac{\pi}{3}$

0.25 a) Write the number a in trigonometric form.

0.5 b) Verify that the affix of the point B image of the point A by the rotation R is $b=2\left(\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right)$

0.5 3) a) We consider the point C of affix $c=1+i$. Show that $b^2-c^2=2\sqrt{3}$

0.5 b) Let t be the translation with vector $\overrightarrow{OC}$, and the point D image of B by the translation t Show that $OD=|b+c|$

0.5 c) Deduce that $OD\times BC=2\sqrt{3}$

Exercise 3: (3 points)

An urn contains 12 balls , indistinguishable by touch, distributed as follows :

3 red balls each carrying the number 1 , and 3 red balls each carrying the number 2 ,

and 6 green balls each carrying the number 2.

We draw , randomly and simultaneously, two balls from the urn , and we consider the following events:

A : " get two balls carrying the same number "

B : " get two balls of different colors "

C : " get two balls carrying two numbers wich sum is equal to three"

1.5 **1) Show that** $p(A)=\frac{13}{22}$ **and** $p(B)=\frac{6}{11}$ **and calculate** $p(C)$

0.5 **2) a) Show that** $p(A\cap B)=\frac{3}{11}$

0.5 **b) Are the events** A **and** B **independent? Justify your answer.**

0.5 **3) Given the event** B **, calculate the probability of obtaining two balls carrying the same number.**

Exercise 4 : (2 points)

0.5 **1) a) Show that the function** $H: x\mapsto xe^x$ **is a primitive of the function** $h: x\mapsto (x+1)e^x$ **on** IR

0.5 **b) Deduce that** $\int_0^1 (x+1)e^x dx = e$

1 **2) Using an integration by parts, calculate** $\int_0^1 (x^2+2x-1)e^x dx$

Problem : (9 points)

I - Let g **be the numerical function defined on** $]0,+\infty[$ **by** $g(x)=x^3-1-2\ln^2 x+2\ln x$

The table beside is the table of variations of the function g **on the interval** $]0,+\infty[$

x	0	$+\infty$
$g'(x)$	+	
$g(x)$	$-\infty$ ↗	$+\infty$

0.25 **1) Calculate** $g(1)$

0.5 **2) From this table , determine the sign of** $g(x)$ **on each of the two intervals** $]0,1]$ **and** $[1,+\infty[$

II - We consider the numerical function defined on the interval $]0,+\infty[$ by

$$f(x)=x-\frac{1}{2}+\frac{1}{2x^2}+\left(\frac{\ln x}{x}\right)^2$$

Let (C) be the curve of f in an orthonormal coordinate system $(O,\vec{i},\vec{j})$

0.5 **1) a)** Verify that $\lim\limits_{x\to+\infty} f(x)=+\infty$

0.5 **b)** Show that the line (D) of equation $y=x-\frac{1}{2}$ is asymptote to the curve (C) at $+\infty$

0.25 **c)** Determine the relative position of the line (D) and the curve (C)

0.75 **2)** Show that $\lim\limits_{\substack{x\to0\\x>0}} f(x)=+\infty$ and interpret geometrically the obtained result.

1 **3) a)** Show that $f'(x)=\frac{g(x)}{x^3}$ for every x on the interval $]0,+\infty[$

0.5 **b)** Show that f is decreasing on the interval $]0\,,1]$ and increasing on the interval $[1\,,+\infty[$

0.5 **c)** Set up the table of variations of the function f on the interval $]0,+\infty[$

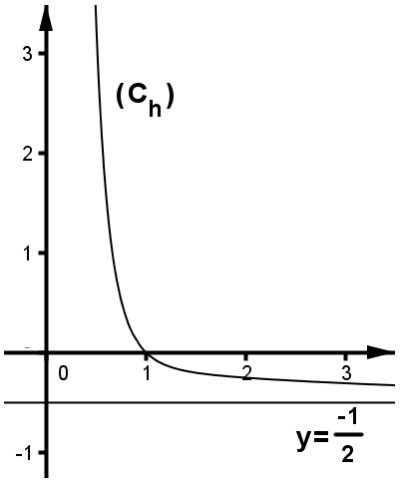
1 **4)** Sketch the line (D) and the curve (C) in the same coordinate system $(O,\vec{i},\vec{j})$ (unit: $1cm$)

III - We consider the numerical function h defined on the interval $]0,+\infty[$ by : $h(x)=f(x)-x$

0.25 **1) a)** Verify that : $h(1)=0$

b) In the figure beside (C_h) is the representative curve of the function h

Determine the sign of $h(x)$ on each of the two intervals $]0,1]$ and $[1,+\infty[$ and then deduce that for every x on the interval $[1\,,+\infty[$, $f(x)\le x$ 0.75



2) We consider the numerical sequence (u_n) defined by:

$u_0=e$ and $u_{n+1}=f(u_n)$ for every natural number n

0.75 **a)** Show by induction that $1\le u_n\le e$ for every natural number n

0.75 **b)** Show that the sequence (u_n) is decreasing. *(you can use the result of the question III-1)b))*

0.75 **c)** Deduce that the sequence (u_n) is convergent and determine its limit.